On Weighted Averages

Darren Tapp

July 7, 2020

Abstract

This article is to compute weighted averages. The intended application is placing orders on a market where the average buy or sell reduces to predictable value.

1 Introduction

We prove a theorem about weighted averages in this paper.

Theorem 1. *Assume g is a real number and let $n \geq 1$ be an integer, then the weighted average of the sequence:*

$$(1, 1+4g, 1+5g, \ldots, 1+(n+2)g)$$

with weights

$$\left(3, \left(\frac{4}{3}\right)^0, \left(\frac{4}{3}\right)^1, \ldots, \left(\frac{4}{3}\right)^{n-2}\right)$$

is $1+(n-1)g$.

Corollary 1. *Let $n \geq 1$ be an integer and $\frac{1}{n+2} > g > 0$, then the average of the sequence:*

$$(1, 1-4g, 1-5g, \ldots, 1-(n+2)g)$$

with weights

$$\left(3, \left(\frac{4}{3}\right)^0, \left(\frac{4}{3}\right)^1, \ldots, \left(\frac{4}{3}\right)^{n-2}\right)$$

is $1-(n-1)g$.

The corollary follows from the theorem by replacing g with $-g$. Note the restrictions on the corollary are not needed for the truth of our statement. The restrictions are needed by an intended application.

2 Partial Geometric Sums

This work will rely on partial geometric sums. That is

Lemma 1. *Let $r > 1$ be a real number and $\ell \geq 0$ be an integer then*

$$\sum_{j=0}^{\ell} r^j = \frac{1 - r^{\ell+1}}{1 - r}.$$

There are several proofs of this statement. An algebraic proof can be obtained with Euclid's algorithm.

3 Proof

We now prove theorem 1.

proof: The theorem is trivial for n = 1. We assume the theorem is true for $n - 1$ and proceed by induction.

The inductive assumption implies

$$3 + \sum_{j=0}^{n-3} \left(\frac{4}{3}\right)^j [(1 + (j+4)g] = \left[3 + \sum_{j=0}^{n-3} \left(\frac{4}{3}\right)^j\right] (1 + (n-2)g).$$

This implies that the weighed sum of the theorem is

$$\frac{\left[3 + \sum_{j=0}^{n-3} \left(\frac{4}{3}\right)^j\right] (1 + (n-2)g) + \left(\frac{4}{3}\right)^{n-2} (1 + (n+2)g)}{3 + \sum_{j=0}^{n-2} \left(\frac{4}{3}\right)^j}.$$

Applying lemma 1 we obtain

$$\frac{\left[3 + \frac{1 - \left(\frac{4}{3}\right)^{n-2}}{1 - \frac{4}{3}}\right] (1 + (n-2)g) + \left(\frac{4}{3}\right)^{n-2} (1 + (n+2)g)}{3 + \frac{1 - \left(\frac{4}{3}\right)^{n-1}}{1 - \frac{4}{3}}}$$

Multiplying top and bottom by $-\frac{1}{3} = (1 - \frac{4}{3})$ yields

$$\frac{\left(-\frac{4}{3}\right)^{n-2} (1 + (n-2)g) - \frac{1}{3} \left(\frac{4}{3}\right)^{n-2} (1 + (n+2)g)}{-\left(\frac{4}{3}\right)^{n-1}}$$

Factoring and distributing,

$$\left(\frac{3}{4}\right)\left(1 + ng - 2g + \frac{1}{3}(1 + ng + 2g)\right)$$

combine like terms,

$$\left(\frac{3}{4}\right)\left(\frac{4}{3} + \frac{4}{3}ng - \frac{4}{3}g\right) = 1 + (n-1)g.$$

Thus the theorem is true.