Base Independent Addition for Parents of Young People

Darren S. Tapp

January 11, 2015

This is intended to be a short introduction to what base-independent arithmetic is and why I am teaching it to young people. As of the writing of this document I only have information from three students to draw from. Teaching is always a dynamic process and I will adapt to new methods as is merited.

The number system used most often is the decimal system which has a base of ten. This means that there are ten symbols and new digits are added to represent numbers larger than ten. Before I explain why base-independent arithmetic is valuable, I ask "Why ten?" The answer is simple, students should be taught base ten so that they can communicate with everyone else that uses base ten.

Even when we want a child to be proficient in English, this does not take away from the value of knowing Spanish, German, or Chinese. In this regard I hope to help young people learn arithmetic in base ten, two, three, four, five, and sixteen.

When performing arithmetic in different bases, I would denote number 23 base 10 as $(23)_{10}$. This will lead to equations like $(6)_8 + (6)_8 = (14)_8$. $(14)_8$ has a 1 in the eights place and a 4 in the ones place, which translates as $8 + 4 = 12$ in decimal/base ten. As you can see this notation system is abstract and complicated, and I believe it is not appropriate for young learners. Nonetheless, the concepts can be easily communicated through adapted teaching methods.

This is one reason you might see me use strange symbols that don't have a context outside of our discussion. Any given day we may invent or recycle one base, and then we relate that to decimal if need be.

One important reason that I believe it is essential to understand different bases is *applications*. Numbers in different bases show up in several contexts. Computers work with binary, which is base 2, and sometimes information is expressed in hexadecimal, which is base 16. Hexadecimal is used because it is able to communicate 8 bits, or one byte, in two digits. A bitcoin address that is a 160 bit number written in base 58. My goal is to have students understand and perform arithmetic in binary, decimal, and hexadecimal. I also teach base three, four, and five, for explanation and fun. It's an instructional tool.

Another important reason for children to understand base-independent arithmetic is that it is the very foundation of mathematics. As someone who teaches math to adults, it seems that people feel as if math has been "imposed" on them. Some students believe they need to go to someone else to recieve instruction in math. The truth is that, once you know the definitions, everything in math can be figured out and learning can become self-directed. There is no need to seek an outside agent to teach math once the general concept is known.

I want students to feel free to be as creative as they can be within the bounds of being correct. At the early stages of math instruction, switching bases allows more creativity without adding to the abstraction level.