Base-4 Addition for Young People

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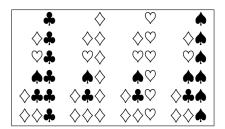
Numbers never stop. No matter how big a number you have, there always is a number that is just one more. To express numbers we will use symbols. We could make symbols look like anything, but it's not possible to keep track of a symbol for every number. So we'll create just a few symbols to start counting. I'll pick \clubsuit , \diamondsuit , \heartsuit , \spadesuit . You can make others.

Next we'll need to put the symbols in order. Here's the order I choose.

*	\Diamond	\Diamond	^	
first	second	third	fourth	
0	1	2	3	

With these four symbols we can count to three. It will be important to have a number that means no objects or zero. Now, how could we write a new number without making up a new symbol? One way is to use a \diamondsuit , a one, to represent one cycle through of all the symbols. We could write this $\diamondsuit \clubsuit$, one cycle and no more. Since there are only four symbols that we cycle through this means four. Now to show one more we could write $\diamondsuit \diamondsuit$ to mean four and one more or five. Then $\diamondsuit \heartsuit$ for four and two more or six. This continues $\diamondsuit \spadesuit$ for seven, then $\heartsuit \clubsuit$ for two four's or eight.

If we continue with this pattern then we can count as follows:



Once we cycle through the second place, or digit, we can then add another place or digit to remember that we cycled through four, four times. Thus the number $\diamondsuit \clubsuit \clubsuit$ stands for four four's or 4+4+4+4 or 16. We repeat the pattern so that the next number is $\diamondsuit \clubsuit \diamondsuit$, then $\diamondsuit \clubsuit \heartsuit$, $\diamondsuit \clubsuit \spadesuit$, $\diamondsuit \diamondsuit \clubsuit$. The number $\diamondsuit \diamondsuit \clubsuit$ is 16+4=20. $\diamondsuit \diamondsuit \spadesuit$ is 16+4+3 which is 23.

Once we understand the numbers we can add(+) them. To add numbers, put them together and count how many you have. For example: $\diamondsuit + \diamondsuit = \heartsuit$, $\heartsuit + \diamondsuit = \spadesuit$, $\spadesuit + \diamondsuit = \diamondsuit \clubsuit$, $\diamondsuit \clubsuit + \spadesuit = \diamondsuit \spadesuit$.

For small numbers, we can make a table that explains addition. This is called an addition table.

+	.	\Diamond	\Diamond	•	\$	$\Diamond \Diamond$
*	4	\Diamond	\Diamond	•	\$	$\Diamond \Diamond$
\Diamond	\Diamond	\Diamond	•	\$	$\Diamond \Diamond$	$\Diamond \heartsuit$
\Diamond	0	•	\$	$\Diamond \Diamond$		$\Diamond \spadesuit$
•	•	\$	$\Diamond \Diamond$	$\Diamond \heartsuit$	$\Diamond \spadesuit$	♡♣
♦ ♣	\$	$\Diamond \Diamond$	$\Diamond \heartsuit$	$\Diamond \spadesuit$	♡♣	$\Diamond \Diamond$
$\Diamond \Diamond$	$\Diamond \Diamond$	$\Diamond \heartsuit$	$\Diamond \spadesuit$	₩	$\Diamond \Diamond$	$\Diamond\Diamond$

If we line our numbers up vertically, we can add by adding one digit at a time. If we end up with anything over \spadesuit we can carry a \diamondsuit and add one to the next digit. Here are some examples:

Here are some for you to try.

$$\begin{array}{c} \diamondsuit \heartsuit \\ + \diamondsuit \diamondsuit \end{array}$$

$$\Diamond \clubsuit \\ + \diamondsuit \heartsuit$$

$$\begin{array}{c} \Diamond \spadesuit \Diamond \\ + \Diamond \Diamond \spadesuit \end{array}$$

$$+ \Diamond \Diamond \Diamond \Diamond \\$$

$$\Diamond\Diamond\Diamond \\ +\Diamond\Diamond\Diamond$$